6.3 Investigating Rational Exponents $x^{\frac{1}{n}}$

1. a) Use the fact that $\sqrt{9}$ = 3 to complete the following.

$\sqrt{9} \times \sqrt{9}$ $3 \times 3 = 9$

b) Use the law of exponents for multiplication to complete the following

$9^{\frac{1}{2}} \times 9^{\frac{1}{2}}$ $9^{\frac{1}{2}+\frac{1}{2}} = 9^1$

c) compare the statements in a) and b). What other mathematical operation does the exponent $\frac{1}{2}$ seem to be equivalent to?

square root

2. Because $2^3 = 8$ (two cubed equals eight), we say that the cube root of 8 is 2, and we write $\sqrt[3]{8} = 2$

a) Use the fact that $\sqrt[3]{8} = 2$ to complete the following.

$\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8} = 2 \times 2 \times 2 = 8$

b) use the law of exponents for multiplication to complete the following.

$8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 8^1$

c) compare the statements in a) and b). What other mathematical operation does the exponent $\frac{1}{3}$ seem to be equivalent to?

$\sqrt[3]{8} = 8^{\frac{1}{3}}$

3. Extend your reasoning to make a generalization about the meaning of $x^{\frac{1}{n}}$ (where $x>0$ and n is a natural number)

$\sqrt[n]{x} = x^{\frac{1}{n}}$

Exponent Law

Exponential form radical form

radical sign $\sqrt{\ }$

$$a^{\frac{1}{n}} \xrightarrow{\text{index}} = \sqrt[n]{a}$$

index: indicates what root you want

Evaluate:

$8^{\frac{1}{3}} = \sqrt[3]{8}$

$9^{\frac{1}{2}} = \sqrt[2]{9}$

Note: when the index is 2 we don't write it, it is understood (square root)

Extend the rule:

numerator is the exponent

$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ OR $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

denominator is the index

Ex 1: Evaluate $125^{\frac{2}{3}}$

Does order matter?

method 1

$= \sqrt[3]{15625}$

$= 25$

method 2

$= (5)^2$

$= 25$

Quotient of bases with Rational Exponents

Ex 2: Evaluate $\dfrac{50^{\frac{1}{2}}}{2^{\frac{1}{2}}}$

Does order matter?

method 1

$\dfrac{\sqrt{50}}{\sqrt{2}}$

$= \dfrac{\sqrt{25}\cdot\sqrt{2}}{\sqrt{2}}$

$= 5$

method 2

$\left(\dfrac{50}{2}\right)^{\frac{1}{2}}$

$(25)^{\frac{1}{2}}$

$= 5$

General Rule:

$$\frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}} = \left(\frac{a}{b}\right)^{\frac{1}{2}} = \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Product of radicals

Ex 3: Evaluate $16^{\frac{1}{3}} \times 4^{\frac{1}{3}}$

Does order matter?

method 1

$\sqrt[3]{16} \times \sqrt[3]{4}$

$\sqrt[3]{4} \times \sqrt[3]{4} \times \sqrt[3]{4}$

$4^{\frac{1}{3}} \times 4^{\frac{1}{3}} 4^{\frac{1}{3}}$

$= 4$

method 2

$(16 \times 4)^{\frac{1}{3}}$

$= (64)^{\frac{1}{3}}$

$= \sqrt[3]{64}$

$= 4$

Alternate Method: create like bases

$(4^2)^{\frac{1}{3}} \times 4^{\frac{1}{3}}$

$= 4^{\frac{2}{3}} \times 4^{\frac{1}{3}}$

$= 4^{\frac{2}{3}+\frac{1}{3}}$

$= 4$

General Rule: $a^{\frac{1}{n}} \times b^{\frac{1}{n}} \times c^{\frac{1}{n}} = \sqrt[n]{a} \times \sqrt[n]{b} \times \sqrt[n]{c} = \sqrt[n]{a \times b \times c}$

Even & Odd Roots

Recall: It is not possible to take the square root of a negative number. Why?

No number multiplied by itself will be negative.

Similarly, you cannot take any even root (2, 4, 6, etc) of a negative number.

With odd roots (3, 5, 7, etc), it is possible to have positive or negative values under the root.

Practice:

$121^{\frac{1}{2}} = \sqrt{121} = 11$

$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

$64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = (4 \times 4 \times 4)^2 = (4)^2 = 16$

$(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = -2$

$32^{-\frac{1}{5}} = \left(\frac{1}{32}\right)^{\frac{1}{5}} = \frac{1}{\sqrt[5]{32}}$

$-81^{\frac{3}{4}} = -\left(\sqrt[4]{81}\right)^3 = -(3)^3 = -27$

$\frac{\sqrt{12}}{\sqrt{8} \times \sqrt{6}} = \frac{\sqrt{6} \times \sqrt{2}}{\sqrt{4} \times \sqrt{2} \times \sqrt{6}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

Ex 4: Simplify then evaluate:

$$\frac{\left(8^{\frac{1}{6}}\right)^7}{8^{\frac{1}{2}} \; 8^{\frac{1}{3}}}$$

ANS

$= 2$

Practice:

Simplify the following expressions:

a) $\dfrac{(m^5)^{-\frac{9}{5}}}{\left(m^{-\frac{3}{2}}\right)^4}$ ANS $\dfrac{1}{m^3}$

b) $\dfrac{m^{\frac{1}{2}} \cdot m^{\frac{3}{2}}}{\left(m^{-2}\right)^{\frac{1}{2}}}$ ANS m^3

Practice:

p 415 # 1, 4, 6,

[9 - 11, 15,17](def),

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